for the vector a, with the eigenvalue given by the Rayleigh quotient

$$\lambda = a^T Ma / a^T Na$$

Acknowledgment

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Comparison of Temperature and Velocity Spectra in a Slightly Heated Turbulent Plane Jet

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Introduction

In their experimental investigation of spectra and cospectra of velocity and temperature fluctuations in a slightly heated turbulent boundary layer, Fulachier and Dumas[1] and Fulachier[2] found that, except in the vicinity of the wall, the spectrum of temperature differed significantly from the spectrum of the longitudinal velocity component, the temperature spectrum being shifted to higher frequencies. A close analogy was found, however, between the spectral distribution of the temperature variance $\overline{\theta^2}$ and the spectral distribution of $\overline{q^2}$ or twice the turbulent kinetic energy ($\overline{q^2} \equiv \overline{u^2} + \overline{v^2} + \overline{w^2}$, u is in the x or longitudinal direction, v is in the y or mean shear direction, w is in the spanwise direction). This analogy is equivalent to one between the autocorrelation of θ and that of the fluctuating velocity vector. Fulachier[3] found that the analogy also worked in the case of turbulent boundary layer downstream of a step change in surface temperature or surface suction.

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The search for an analogy between $\overline{q^2}$ and $\overline{\theta^2}$, both of which are scalar quantities, is more attractive if not more relevant than the analogy between one component, usually $\overline{u^2}$, of the Reynolds stress tensor and $\overline{\theta^2}$. From a point of view of transport equations, Corrsin[4] had earlier noted the analogy that existed between the equations for $\overline{q^2}$ and $\overline{\theta^2}$, apart from the appearance of the pressure term in the $\overline{q^2}$ equation. Equations for two-point correlations of temperature and of the fluctuating velocity vector[2] more closely reflect this analogy, at least in the case of homogeneous turbulence when the term containing the pressure fluctuations disappears.

Fulachier and Dumas[1] concentrated especially on the low-frequency range that accounts for the major part of $\overline{q^2}$ and $\overline{\theta^2}$ in a boundary layer. In this Note, we seek to establish the validity of the analogy in the self-preserving region of a free shear flow: a turbulent plane jet. It is pertinent to point out that the boundary conditions for the velocity and thermal fields differ in an important way between a flow over a heated wall and a heated free shear flow. The presence of the wall in the boundary layer ensures a close link, near the wall, between spectra of u and θ fluctuations at every streamwise location in the flow. Such a link is absent in a jet.

Experimental Arrangement and Conditions

Velocity fluctuations u and v and the temperature fluctuation θ were measured with an X-probe/cold-wire arrangement at a distance $x=40d$ ($d=12.7$ mm is the nozzle width; the nozzle height is 250 mm) from the nozzle exit. The hot wires (5 μm Pt-10% Rh, 0.6-mm long) were separated by about 0.5 mm. The 0.63-μm Pt-10% Rh cold wire (0.6-mm long) was positioned about 0.5 mm upstream of the center of the X probe, orthogonally to the X probe. For the measurements of w, the cold wire was removed and the X probe, rotated so that the wires were in the x-z plane, was used in the unheated jet. The hot wires were operated with constant temperature anemometers at an overheat ratio of 1.8 while the cold wire was operated with a constant current (0.1 mA) circuit. Details of the apparatus and experimental procedure are given in Ref. 5. Hot- and cold-wire voltages were digitized for a duration of approximately 60 s, at a sampling frequency of 333 Hz into a PDP 11/34 computer. The hot-wire voltages, for the measurement of u and v fluctuations, were linearized on the computer following the removal of the temperature contamination. The velocity sensitivity of the cold wire was negligible.

The nominal jet speed U_j was 9 ms^{-1} and the nominal jet temperature, relative to ambient, was 25 K. At $x/d=40$, the mean velocity U_0 at the jet centerline was about 3.4 m/s and the velocity half-width L_u was 0.6 m. Self-preservation of the jet was established at $x/d \simeq 20$ from mean velocity and mean temperature profiles, distributions of Reynolds stresses and heat fluxes.

Experimental Results and Discussion

Spectra computed using a fast Fourier transform algorithm are presented for two values of the ratio $\eta \equiv y/L_u$ (0 and 0.5), y being measured from the jet centerline. The value of 0.5 for η corresponds to a location where the kinematic Reynolds shear stress $\overline{uv}$ and the thermometric heat flux $\overline{v\theta}$ are relatively large, although not quite maximum. The flow is fully turbulent (the intermittency factor is unity) for values of η extending up to about 1.

The spectral density F_α is defined such that

$$\int_0^\infty F_\alpha(\omega^*)\mathrm{d}\omega^* = 1$$

with $\omega^* = \omega L_u / U_0$ where ω is the circular frequency. The subscript α stands for u, v, w, θ, or q. For convenience, we

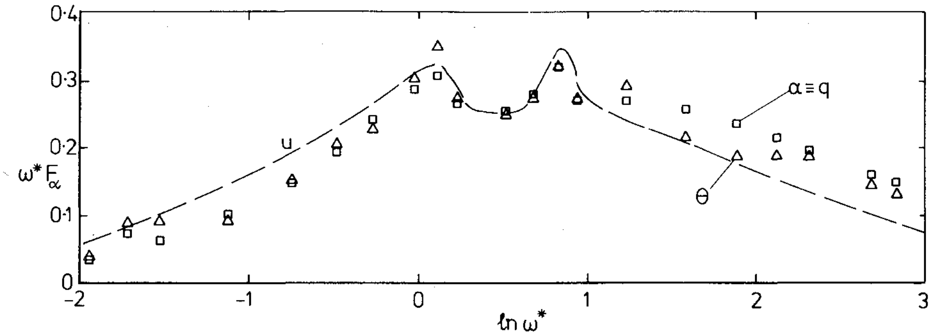

Fig. 1 Spectral analogy between θ^2 and q^2 at $x/d=40$ ($\eta=0$).

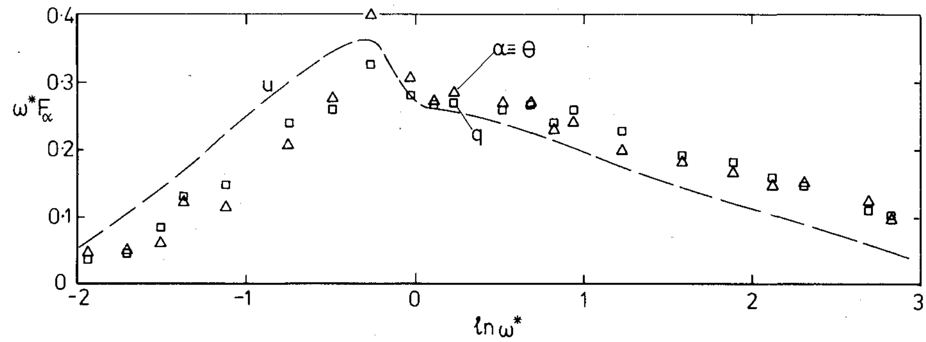

Fig. 2 Spectral analogy between θ^2 and q^2 at $x/d=40$ ($\eta=0.5$).

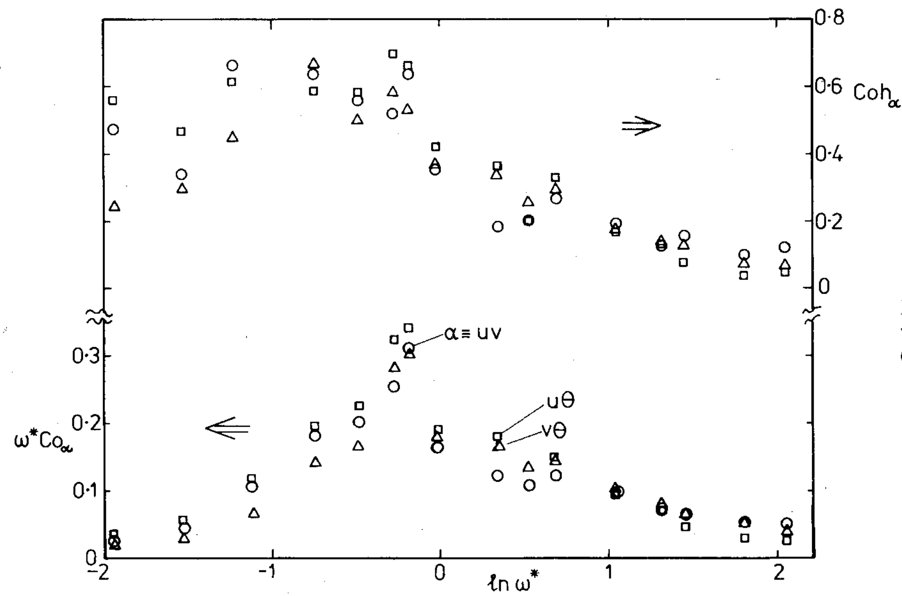

Fig. 3 Cospectra and coherences between velocity and temperature fluctuations at $x/d=40$ ($\eta=0.5$); $\rho_{uv}=0.52$, $\rho_{v\theta}=0.48$, $\rho_{u\theta}=0.56$.

denote by F_q the spectral distribution of q^2

$$F_q=\frac{\overline{u^2}}{\overline{q^2}}F_u+\frac{\overline{v^2}}{\overline{q^2}}F_v+\frac{\overline{w^2}}{\overline{q^2}}F_w$$

Distribution of $\omega^* F_q$ and $\omega^* F_\theta$ are in good agreement with each other at both $\eta=0$ (Fig. 1) and $\eta=0.5$ (Fig. 2). This agreement is perhaps not surprising in view of the reasonably close similarity between spectra of u, v, and w, especially at $\eta=0$. Nevertheless, it is clear that F_q is more closely analogous to F_θ than is F_u. It should also be noted that the similarity between F_q and F_θ is reasonably well established in a spectral region which makes a high contribution (Figs. 1 and 2) to q^2 and θ^2. Speculatively, this result may, in part, reflect the influence of coherent structures whose passage frequency occurs at $\omega^* \simeq 0.7$ (e.g., Cervantes and Goldschmidt[6]).

It is of interest, notwithstanding our previous reservations about the analogy between vector and scalar fields, to briefly comment on the behavior of cospectra and coherences between velocity and temperature fluctuations in the present flow. Fulachier[2] noted that the peak of the temperature production cospectrum was shifted to higher frequencies vis-à-vis that of the velocity production cospectrum. Chevray and Tutu[7] compared filtered correlations between u and v and those between v and θ obtained at $\eta \simeq 0.8$ in a circular jet. They concluded that small-scale turbulent motions were more efficient in transporting heat than in transporting momentum. Tavoularis and Corrsin[8] found that values of the narrowband passed turbulent shear stress and heat transport coherences in their quasihomogeneous shear flow decreased as the bandpass wavenumber increased. As this wavenumber approached the dissipation end of the cospectrum, the heat transport coherence remained larger than the shear stress coherence. This behavior, similar to that observed by Fulachier and Dumas[1] and Chevray and Tutu[7] in nonhomogeneous shear flows, seems to support the speculation[8] that temperature fluctuations show larger departures from local isotropy than velocity fluctuations.

Cospectral values for the combinations u-v, v-θ, and also u-θ are shown in Fig. 3 for $x/d = 40$ and for $\eta \simeq 0.5$. The cospectrum Co is defined such that the area under the distributions is equal to the correlation coefficient $\rho_{\alpha\beta}$ ($\equiv \overline{\alpha\beta} / \overline{\alpha^2}^{1/2}\,\overline{\beta^2}^{1/2}$, where α, β stand for u, v, or θ). All three cospectra exhibit a maximum at approximately the same frequency ($\ell n\omega^* \simeq -0.2$). For frequencies smaller than this peak frequency, Co_{uv} is larger than $Co_{v\theta}$ whereas $Co_{v\theta}$ is slightly larger than Co_{uv} at higher frequencies. This trend is in reasonable agreement with that previously reported for other flows but there is practically no difference (Fig. 3) between the coherences Coh_{uv} and $Coh_{v\theta}$ [note, $Coh_{\alpha\beta} = (Co^2 + Q^2)/(F_\alpha F_\beta)$, where Q is the quadrature spectrum] for frequencies greater than that at which the cospectra are maximum.

Conclusions

The analogy between F_θ and F_q, introduced and verified for the case of a slightly heated turbulent boundary layer,[1] applies to the self-preserving region of a turbulent plane jet. In this region, the spectrum of v differs from other velocity spectra but this difference, especially near the jet centerline, is much less marked than in the boundary layer.[2] In the jet, the cospectra and, in particular, the coherences between longitudinal and lateral velocity fluctuations do not differ appreciably from those between temperature and the lateral velocity fluctuations. This difference is much more pronounced in the boundary layer.[1] In this latter flow, a close liaison is maintained, close to the heated wall, between F_u and F_θ. The temperature spectrum becomes progressively more dependent on the spectrum of v as the distance from the wall increases. In the jet, the temperature spectrum depends on all three velocity fluctuations, especially near the centerline.

Acknowledgment

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Chemistry of Combustion of Double-Base Propellants Through Sliver Analysis

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Introduction

INTEREST in systematic study to understand the combustion mechanism of double-base propellants (DBP), thrives because the existing literature is inadequate to explain many experimental observations. Some detailed investigations on the combustion mechanism of composite propellants have been carried out in recent years in which attempts were made to understand the condensed-phase combustion mechanism through intermediary analysis.[1,2] The result strongly supports the occurrence of exothermic condensed-phase reactions.

In the present investigation an attempt has been made to show evidence of the occurrence and nature of condensed-phase reactions during combustion in double-base propellants through sliver analysis. The sliver sample is the self-extinguished propellant sample obtained from the actual motor firing. It is the belief that some clarification of the decomposition process in the surface layer of burning propellants can be made by examination of the surface material from the quenched samples.

The in situ burning surface (sliver sample) was obtained by natural flame extinguishment. The procedure is outlined in the next section. In the present work it is assumed that the sliver represents the true burning surface. It is believed that the analysis of this sliver sample will reveal the nature of chemical reactions occurring during the combustion of DBPs.

Experimental Procedure

The composition of the DBP used was: 26.5% nitroglycerine (NG), 57.5% nitrocellulose (NC), 5.0% dibutylphthalate, 6.0% dinitrotoluene (DNT)/trinitrotoluene (TNT), and 3.5% 2-nitrodiphenyl amine (2-nDPA) and the rest inorganic salts.

A 30 ton thrust motor was filled with 14 tubular uninhibited DBP propellant grains that burned from inside to outside as well as outside to inside. Setting the nozzle to a required pressure (70 kg·cm^{-2}), the motor was fired to evaluate the ballistic parameters. As the propellant burns, the web thickness decreases and contracts to such an extent that under the pressure the grain crumbles and the unburned pieces are thrown out. As the pieces come out of the nozzle, the burning stops because of a sudden drop in pressure. The pieces thus ejected from the motor (technically called as "slivers") were collected. The top layer (50 μm) of the sliver surfaces was scratched gently with a jewelery file to collect a dark brownish residue for analysis.

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